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Asymmetric unbiased fluctuations are sufficient for the operation of a correlation ratchet

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Abstract

We study correlation ratchets with mean-zero (unbiased) nonequilibrium noise with a nonvanishing correlation function of odd order greater than one. Asymmetric noise of this type can induce a subtle bias into nonequilibrium systems which can interact with other biasing influences, such as a spatial asymmetry, in a complicated way. Since temporal asymmetry has to be regarded as a generic property of nonequilibrium systems, these effects are expected to be ubiquitous in nature.

A number of recent attempts to understand broad principles of energy transduction in nonequilibrium physical and biological systems have focused on correlation ratchets – systems which extract work out of fluctuations which are correlated in time (see Refs. [1–7]). It has been shown that the combination of a broken spatial symmetry in the potential (or ratchet potential) and time correlations in the driving are crucial, and enough to allow the transformation of the fluctuations into work. Here we show that a broken spatial symmetry is *not required*, and that temporally asymmetric fluctuations (with mean zero) can be used to do work, *even when the ratchet potential is completely symmetric*. Temporal asymmetry, defined as a nonvanishing of the odd moments or correlation functions of the fluctuations order higher than one, and is clearly a generic property of the nonequilibrium fluctuation [8–10].

Here we consider the simplest imaginable system which contains the crucial elements. We strongly emphasize here that it is not the model or its simplifying assumptions, but the physical principle of temporal asymmetry that we illustrate which is important. We hope that the extension to more complicated and realistic situations will present themselves naturally to the mind of the attentive reader.

We consider an over-damped particle in a periodic potential $U(x) = U(x+\lambda)$. The motion of the particle in the periodic potential obeys

$$\dot{x} + U'(x) = \zeta(t) + F(t), \tag{1}$$

where $\zeta(t)$ is the thermal white noise with $\langle\zeta(t)\rangle = 0$, $\langle\zeta(t)\zeta(s)\rangle = 2kT\delta(t-s)$, and $F(t)$ is some external driving force. The evolution of the probability density for x is given by the associated Fokker–Planck equation,

$$\partial_t \rho(x,t) = \partial_x(\Phi'(x,t) + kT\partial_x)\rho, \tag{2}$$

where $\Phi'(x,t) = U(x) - F(t)x$.

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The steady-state solution (for constant F) is found by imposing periodic boundary conditions $\rho_s(x) = \rho_s(x+\lambda)$, and normalization $\int_0^\lambda \rho_s(x)\,dx = 1$ [11]. This yields an exact expression for the mean rate of change of x,

$$\langle \dot{x} \rangle_t = \frac{1 - e^{-\lambda F/kT}}{(1/kT) \oint dy \oint dx\, e^{-\Phi(y)/kT} e^{\Phi(x+y)/kT}}. \tag{3}$$

For $F(t)$ which changes on time scales much slower than the principal relaxation time of the system, the net voltage is found by averaging,

$$\langle \dot{x} \rangle = \lim_{T\to\infty} \frac{1}{T} \int_0^T \langle \dot{x} \rangle_t \, dt.$$

Since the effect is typically exponential in the applied force, a force with mean zero can give rise to a net transport if the force is applied asymmetrically in time. Since $\langle \dot{x}(F) \rangle$ is an antisymmetric nonlinear function of F, it can be expanded in a series in the odd moments of $F(t)$,

$$\langle \dot{x} \rangle = \sum_{n=1}^{\infty} c_{2n+1} \langle F^{2n+1}(t) \rangle. \tag{4}$$

Therefore there will be a net transport whenever any odd moment $\langle F^{2n+1}(t) \rangle \neq 0$. This happens *even though the net current is zero*, and therefore is a fluctuation-induced effect. For faster noise this statement can be generalized to the case where an odd correlation function of order greater than one is nonvanishing.

As a simple example of the effects of slow noise we consider the following exactly solvable case. In this case $U(x)$ is the piecewise linear potential pictured in Fig. 1a, as considered in Ref. [1]. The potential is periodic and extends to infinity in both directions. λ measures the spacing of the wells, λ_1 and λ_2 the inverse steepnesses of the potential in opposite directions out of the wells, and Q the well depths. The expression for the current in the adiabatic limit, which measures the work done by the ratchet has already been derived in this case [1,11], where

$$J(F) = \frac{P_2^2 \sinh(\lambda F/2kT)}{kT(\lambda/Q)^2 P_3 - (\lambda/Q) P_1 P_2 \sinh(\lambda F/2kT)}, \tag{5}$$

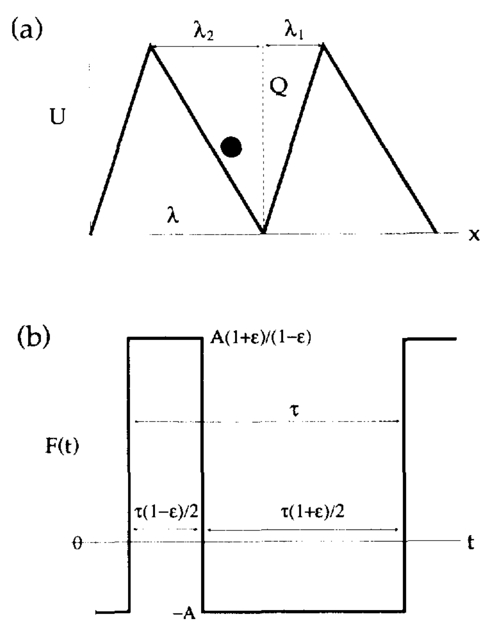

Fig. 1. (a) The simple piecewise ratchet potential, where the spatial degree of asymmetry is given by the parameter $\delta = \lambda_1 - \lambda_2$. (b) The driving force $F(t)$ which preserved the zero mean $\langle F(t) \rangle = 0$, where the temporal asymmetry is given by the parameter ϵ.

$$P_1 = \delta + \frac{\lambda^2 - \delta^2}{4} \frac{F}{Q},$$

$$P_2 = \left(1 - \frac{\delta F}{2Q}\right)^2 - \left(\frac{\lambda F}{2Q}\right)^2, \tag{6}$$

$$P_3 = \cosh[(Q - \tfrac{1}{2}\delta F)/kT] - \cosh(\lambda F/2kT), \tag{7}$$

where $\lambda = \lambda_1 + \lambda_2$ and $\delta = \lambda_1 - \lambda_2$. The average current, the quantity of primary interest, is given by

$$\langle J \rangle = \frac{1}{T} \int_0^T J(F(t))\, dt, \tag{8}$$

where τ is the period of the driving force $F(t)$, which is assumed longer than any other time scale of the system in this adiabatic limit. Magnasco considered this case, but only for $F(t)$ symmetric in time, $F(t) = F(n\tau - t)$. Here we will again consider a driving with a *zero mean*, $\langle F(t) \rangle = 0$, but which is asymmetric in time,

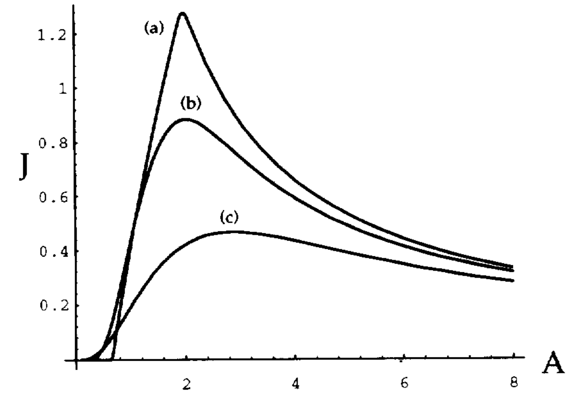

Fig. 2. The current versus A for a symmetric potential ($\delta = 0$), with $Q = 1$, $\lambda = 1$, $\epsilon = \frac{1}{2}$ and (a) $kT = 0.01$, (b) $kT = 0.1$, (c) $kT = 0.3$.

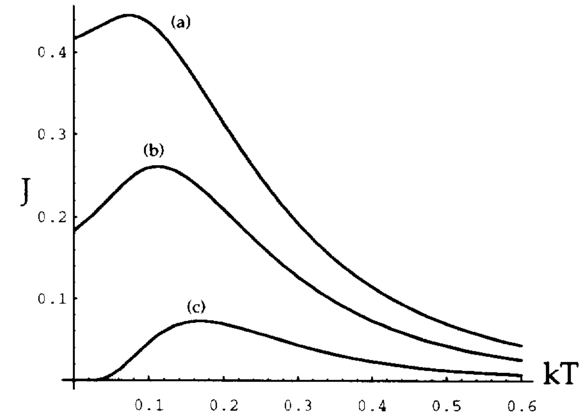

Fig. 3. The current versus kT for a symmetric potential ($\delta = 0$), with $Q = 1$, $\lambda = 1$, $\epsilon = \frac{1}{2}$ and (a) $A = 1$, (b) $A = 0.8$, (c) $A = 0.5$.

$$F(t) = \frac{1+\epsilon}{1-\epsilon}A, \quad 0 \leqslant t < \tfrac{1}{2}\tau(1-\epsilon), \quad \text{mod}\ \tau,$$
$$= -A, \qquad \tfrac{1}{2}\tau(1-\epsilon) > t \leqslant \tau, \quad \text{mod}\ \tau, \quad (9)$$

as shown in Fig. 1b. In this case the time averaged current is easily calculated,

$$\langle J \rangle = \tfrac{1}{2}[(1+\epsilon)J(A) + (1-\epsilon)J(-(1+\epsilon)A/(1-\epsilon))]. \qquad (10)$$

Fig. 2 shows that the current is a peaked function of the amplitude of the driving. Thus, everything else being constant, there is an optimal amplitude for the driving. Similarly Fig. 3 shows that the current is also a peaked function of kT. The driving, the potential, and the thermal noise in fact play cooperative roles. Unless A is quite large, there are no transitions out of the wells when $kT = 0$, and therefore no current, but if the noise is too large it washes out both the potential and the details of the driving, and the current again goes to zero. Similarly, without the driving the transitions in either direction are the same, but if the driving is too large the potential plays less of a selective role, and the current drops back down. Here the main features introduced by the temporal asymmetry are the interplay of the lower potential barriers in the positive direction relative to the negative direction (for this particular driving) and the corresponding shorter and longer times respectively the force is felt. These types of competitive effects appear ubiquitously in systems where there is an interplay between thermal activation and dynamics.

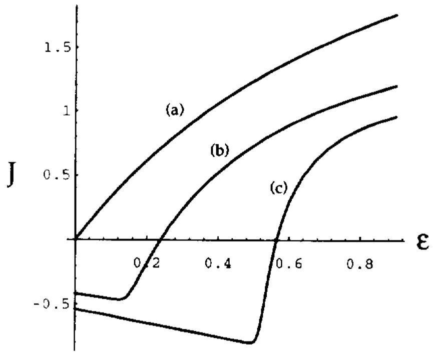

Fig. 4. The current versus the time asymmetry parameter ϵ with $Q = 1$, $\lambda = 1$, $kT = 0.01$, and $A = 2.1$, for (a) a symmetric potential $\delta = 0$ and asymmetric potentials (b) $\delta = -0.3$, (c) $\delta = -0.7$.

Figs. 2 and 3 are for completely symmetric potentials. In these cases the exponential Arrhenius dependence of the thermal activations over the barriers overcomes the time factors, and the current is in the opposite direction of that which is produced by a temporal symmetry and a spatial asymmetry. This effect is of course reversed if $\epsilon \to -\epsilon$. There can also be competitive effects between the temporal asymmetry and the spatial asymmetry, as pictured in Fig. 4, which gives rise to a very unusual switching of the direction of the current as the asymmetry factor ϵ is varied. This reversal represents the competition of the spatial asymmetry, which dominates for small ϵ, and the temporal asymmetry, which dominates for large ϵ.

The reasons for the phenomena we discuss here are not related to the specific approximations which allowed for an analytic solution in our very simple illustration, but are ubiquitous characteristics of nonequilibrium fluctuations, and deserve to be studied in more detail. The main idea here, that of a temporal asymmetry in the driving, can be easily incorporated by extending the theory of Ref. [2] to continuous, but non-Gaussian noise, and the discrete noise models of Ref. [5], with qualitatively similar results. In addition, fluctuating potential models [3,4] can be made to incorporate temporal asymmetry as an important factor if the potential changes shape as it fluctuates in magnitude.

Our main point here is that temporal asymmetry, defined as nonvanishing of the odd moments higher than first order, can be expected to be a ubiquitous property of most nonequilibrium systems, and can give rise to nonequilibrium transport. We believe it is a fundamental principle, equal in importance to the observation that spatial asymmetry and temporal correlations are sufficient for nonequilibrium transport.

Time correlations in the driving will influence the transitions in either direction. Since this influence depends on the shape of the potential, as well as on the properties of the noise, an asymmetry in the potential will give rise to a net current. However, when the noise is temporally asymmetric, its correlation properties *in either direction are different, and a net current can arise even in the absence of a spatial asymmetry.* Note that the dependence of the strength and direction of the current on the properties of the noise discussed here is not related to the similar results of Refs. [2,5], in which only temporally symmetric fluctuations where considered, and the current vanished in the spatially symmetric case.

Such temporal asymmetry is present in virtually any waveform or noise with a nontrivial distribution of frequency component phases. It is believed that many driven biochemical processes work by cycling through a sequence of intermediary states. These cycles are driven by the steps in the hydrolysis of ATP. If the steps in this sequence have different time scales, which is generally the case, the result will be temporal asymmetry, as well as correlations. Unless there is a physical (generally equilibrium) restriction, temporal asymmetry will probably be ubiquitous in nonequilibrium systems, such as biological energy transducers.

The reader should take note of another application of temporal asymmetry by Mahato and Jayannavar [10]. There are a number of interesting applications of nonequilibrium fluctuations which make use of both asymmetric fluctuations, and asymmetry in the potential. One which we think is particularly interesting is "nonequilibrium kinetic focusing", in which a complicated, multistable system can be selectively focused into a thermally inaccessible state by driving the system with nonequilibrium fluctuations [12]. The focusing is made possible by tuning the noise parameters to enhance transport into a given state, and suppress transport out of that state. In a multistate model of the Shaker K^+ ion channel we have shown that the probabilities of a thermally inaccessible state can be increased in this way from near zero under optimal static conditions to near one. We hope that ideas such as this will find applications in experimental biology and chemistry.

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